

## Synopsis – Grade 10 Math Term I

### Chapter 1: Real Numbers

❖ **Euclid’s division lemma:**

For any given positive integers  $a$  and  $b$ , there exists unique integers  $q$  and  $r$  such that  $a = bq + r$  where  $0 \leq r < b$

- If  $b$  divides  $a$ , then  $r = 0$

Example: For  $a = 15$ ,  $b = 3$ , it can be observed that

$$15 = 3 \times 5 + 0$$

Here,  $q = 5$  and  $r = 0$

- If  $b$  does not divide  $a$ , then  $0 < r < b$

Example: For  $a = 20$ ,  $b = 6$ , it can be observed that  $20 = 6 \times 3 + 2$

Here,  $q = 3$ ,  $r = 2$ ,  $0 < 2 < 6$

❖ **Results on the basis of Euclid’s division lemma:**

- Every positive even integer is of the form  $2q$ , while every positive odd integer is of the form  $2q + 1$ , where  $q$  is some integer.
- Any positive integer is of the form  $3q$ ,  $3q + 1$  or  $3q + 2$ , where  $q$  is an integer.

❖ Euclid’s division algorithm is a series of well-defined steps based on “Euclid’s division lemma”, to give a procedure for calculating problems.

❖ **Finding HCF of two positive integers  $a$  and  $b$  ( $a > b$ ) by using Euclid’s division algorithm:**

**Step 1:** Applying Euclid’s division lemma to  $a$  and  $b$  to find whole numbers  $q$  and  $r$ , such that

$$a = bq + r, 0 \leq r < b$$

**Step 2:** If  $r = 0$ , then  $\text{HCF}(a, b) = b$

If  $r \neq 0$ , then again apply division lemma to  $b$  and  $r$

**Step 3:** Continue the same procedure till the remainder is 0. The divisor at this stage will be the HCF of  $a$  and  $b$ .

**Note:**  $\text{HCF}(a, b) = \text{HCF}(b, r)$

❖ **Fundamental theorem of arithmetic**

Every composite number can be uniquely expressed (factorised) as a product of primes apart from the order in which the prime factors occur.

**Example:** 1260 can be uniquely factorised as

2	1260
2	630

3	315
3	105
5	35
	7

$$1260 = 2 \times 2 \times 3 \times 3 \times 5 \times 7$$

- ❖ **Result:** For any positive integer  $a, b$ ,  $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$
- ❖ **Application of Fundamental theorem of arithmetic**

**Example:** Check whether  $15^n$  is divisible by 10 or not for any natural number  $n$ . Justify your answer.

**Solution:**

A number is divisible by 10 if it is divisible by both 2 and 5.

$$15^n = (3 \times 5)^n$$

3 and 5 are the only primes that occur in the factorisation of  $15^n$ .

By uniqueness of fundamental theorem of Arithmetic, there is no other prime except 3 and 5 in the factorisation of  $15^n$ .

2 does not occur in the factorisation of  $15^n$ .

Hence,  $15^n$  is not divisible by 10.

- ❖ **Rational numbers**

Every rational number can be expressed in the form  $\frac{p}{q}$ , where  $p, q$  are integers  $q \neq 0$

Example,  $1 = \frac{1}{1}, 0 = \frac{0}{q}$  where  $q \neq 0$  is any integer

$$1.2 = \frac{12}{10} = \frac{6}{5}$$

- ❖ **Irrational numbers**

Every irrational number cannot be expressed in the form  $\frac{p}{q}$ , where  $p, q$  are integers and  $q \neq 0$

Example:  $\sqrt{3}, \sqrt{11}, \sqrt{12}$ , etc.

- ❖ **Theorem:** If a prime number  $p$  divides  $a^2$ , then  $p$  divides  $a$ , where  $a$  is a positive integer.
- ❖ **Some results related to the decimal expansion of rational numbers**

- Let  $x$  be a rational number whose decimal expansion terminates.

Then ' $x$ ' can be written in the form  $x = \frac{p}{q}$ , where  $p$  and  $q$  are co-primes and the

prime factorisation of  $q$  is of the form  $2^n 5^m$ , where  $n, m$  are non-negative integers.

- Let  $x = \frac{p}{q}$  be a rational number such that  $q = 2^n 5^m$ , where  $n, m$  are non-negative integers. Then, the decimal expansion of  $x$  terminates.
- If  $x = \frac{p}{q}$  is a rational number such that the prime factorisation of  $q$  is not of the form  $2^n 5^m$ , where  $n, m$  are non negative integers, then  $x$  has a decimal expansion which is non-terminating repeating.

## Chapter 2: Polynomials

### ❖ Polynomial

A polynomial in variable ' $x$ ' is of the form,  $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ , where  $a_n, a_{n-1}, \dots, a_1, a_0$  are constants and ' $n$ ' is a positive integer.

For example,  $p(x) = 4x^3 + 3x - 1$ ,  $g(t) = \sqrt{7}t^3 + t^2 - 8$

### ❖ Degree of a polynomial

The highest power of  $x$  in a polynomial  $p(x)$  is called the degree of the polynomial  $p(x)$ .

- A polynomial of degree 1 is called a linear polynomial.

**Example:**  $f(y) = 2y - 7$

- A polynomial of degree 2 is called a quadratic polynomial.

**Example:**  $g(t) = 2t^2 - 3t + 8$

- A polynomial of degree 3 is called a cubic polynomial.

**Example:**  $f(u) = u^3 - 5u^2 + 10$

- A polynomial of degree 0 is called a constant polynomial.

**Example:**  $h(t) = -\frac{8}{3}$

- The constant polynomial,  $f(x) = 0$ , is called zero polynomial.
- Degree of zero polynomial is not defined.

### ❖ Zeroes of a polynomial

A real number ' $k$ ' is a zero of a polynomial  $p(x)$ , if  $p(k) = 0$ . In this case, ' $k$ ' is also called the root of the equation,  $p(x) = 0$ .

**Note:** A polynomial of degree  $n$  can have at most  $n$  zeroes.

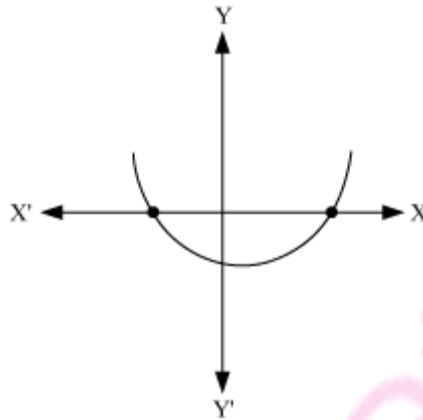
**Example:** 2 and  $-3$  are the zeroes of the quadratic polynomial,  $x^2 + x - 6$ .

$$\left[ \because 2^2 + 2 - 6 = 0, (-3)^2 + (-3) - 6 = 0 \right]$$

❖ **Geometrical meaning of zeroes of a polynomial**

The zero of a polynomial,  $y = p(x)$ , (if it exists) is the  $x$ -coordinate of the point where the graph of  $y = p(x)$  intersects the  $x$ -axis.

**For example:**



In the above graph, the graph intersects the  $x$ -axis at exactly two points.

$\therefore$  The number of zeroes of the corresponding polynomial is 2.

❖ **Relationship between zeroes and coefficients of a polynomial**

• **Linear Polynomial**

The zero of the linear polynomial,  $ax + b$ , is  $\frac{-b}{a} = \frac{-(\text{Constant term})}{\text{Coefficient of } x}$

• **Quadratic polynomial**

If  $\alpha$  and  $\beta$  are the zeroes of the quadratic polynomial,  $p(x) = ax^2 + bx + c$ , then  $(x - \alpha), (x - \beta)$  are the factors of  $p(x)$ .

$$p(x) = ax^2 + bx + c = k[x^2 - (\alpha + \beta)x + \alpha\beta], \text{ where } k \neq 0 \text{ is constant}$$

$$\text{Sum of zeroes} = \alpha + \beta = \frac{-b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

• **Cubic polynomial**

If  $\alpha, \beta, \gamma$  are the zeroes of the cubic polynomial,  $f(x) = ax^3 + bx^2 + cx + d$ , then  $(x - \alpha), (x - \beta), (x - \gamma)$  are the factors of  $f(x)$ .

$$f(x) = ax^3 + bx^2 + cx + d = k[x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma]$$

, where  $k$  is a non-zero constant

$$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\alpha\beta\gamma = \frac{-d}{a} = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$$

#### ❖ Division algorithm for polynomials

It states that for any two polynomials,  $p(x)$  and  $g(x) \neq 0$ , there exists polynomials,  $q(x)$  and  $r(x)$ , such that

$$p(x) = g(x)q(x) + r(x), \text{ where } r(x) = 0 \text{ or degree } r(x) < \text{degree } g(x)$$

Here,  $p(x)$  is called dividend,  $g(x)$  is called divisor,  $q(x)$  is called quotient, and  $r(x)$  is called remainder.

**Example:** Divide  $x^4 - x^3 + 3x^2 - x + 3$  by  $x^2 - x + 1$  and verify the division algorithm.

**Solution:**

It is given that,

$$\text{Dividend} = x^4 - x^3 + 3x^2 - x + 3, \text{ Divisor} = x^2 - x + 1$$

$$\begin{array}{r} \phantom{x^2 - x + 1} \overline{) x^4 - x^3 + 3x^2 - x + 3} \\ \underline{x^4 - x^3 + x^2} \phantom{- x + 3} \\ \phantom{x^4 - x^3 +} 2x^2 - x + 3 \\ \underline{2x^2 - 2x + 2} \phantom{+ 3} \\ \phantom{x^4 - x^3 + 2x^2 -} x + 1 \end{array}$$

Divisor  $\times$  Quotient + Remainder

$$\begin{aligned}
 &= (x^2 - x + 1)(x^2 + 2) + x + 1 \\
 &= x^4 + 2x^2 - x^3 - 2x + x^2 + 2 + x + 1 \\
 &= x^4 - x^3 + 3x^2 - x + 3 = \text{Dividend} \\
 \therefore &\text{ Division algorithm is verified.}
 \end{aligned}$$

### Chapter 3: Pair of Linear Equations in Two Variables

#### ❖ Linear equation in two variables

Linear equation in two variables  $x$  and  $y$  is of the form  $ax + by + c = 0$ , where  $a$ ,  $b$ , and  $c$  are real numbers, such that both  $a$  and  $b$  are not zero.

**Example:**  $6x + 3y = 9$

- A linear equation in two variables has infinitely many solutions.
- Equations of  $x$ -axis and  $y$ -axis are respectively  $y = 0$  and  $x = 0$ .
- The graph of  $x = a$  is a straight line parallel to the  $y$ -axis, and is at a distance of ' $a$ ' units from the  $y$ -axis.
- The graph of  $y = a$  is a straight line parallel to the  $x$ -axis, and is at a distance of ' $a$ ' units from the  $x$ -axis.
- Every point on the graph of a linear equation in two variables is a solution of the linear equation and vice versa.

Example: Consider the linear equation  $6x + y = 12$  ... (1)

(1, 6) is a solution of (1) [LHS =  $6 \times 1 + 6 = 6 + 6 = 12 = \text{RHS}$ ]

But (2, 3) is not a solution of (1) since LHS =  $6 \times 2 + 3 = 12 + 3 = 15 \neq \text{RHS}$

Point (1, 6) lies on the line representing the equation (1), whereas point (2, 3) does not lie on the line.

#### ❖ Pair of linear equation in two variables

- Two linear equations in the same two variables are called a pair of linear equations in two variables.
- The general form of a pair of linear equations is  $a_1x + b_1y + c_1 = 0$ ,  $a_2x + b_2y + c_2 = 0$ , where  $a_1, a_2, b_1, b_2, c_1, c_2$  are real numbers such that  $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$

#### ❖ Graphical representation of linear equations:

Example: Represent the following system of linear equations graphically.

$$x + y + 2 = 0, 2x - 3y + 9 = 0$$

#### **Solution:**

The given equations are

$$x + y + 2 = 0 \quad (1)$$

$$2x - 3y + 9 = 0 \quad (2)$$

Table for the equations  $x + y + 2 = 0$

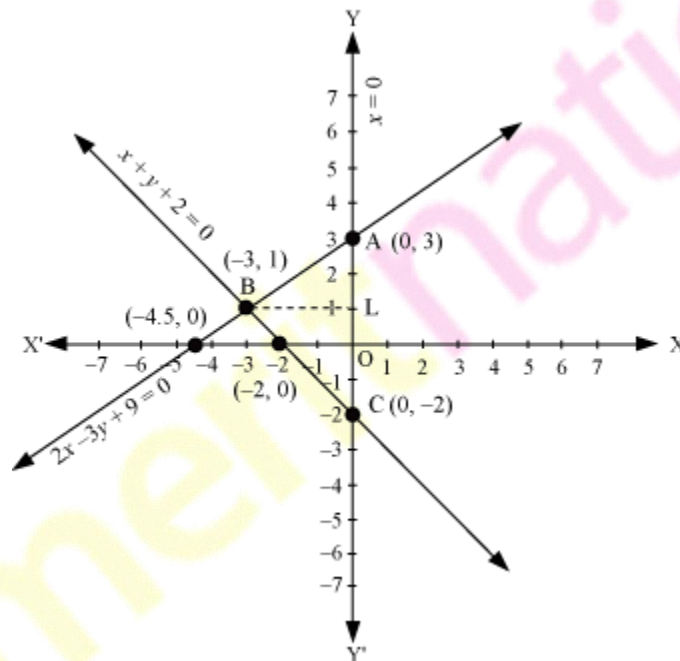
$x$	0	-2
$y$	-2	0

Table for the equation  $2x - 3y + 9 = 0$

$x$	0	-4.5
$y$	3	0

By plotting and joining the points (0, -2) and (-2, 0), the line representing equation (1) is obtained.

By plotting and joining the points (0, 3) and (-4.5, 0), the line representing equation (2) is obtained.



#### ❖ System of simultaneous linear equations

- **Consistent system**

A system of simultaneous linear equations is said to be consistent if it has at least one solution.

- **Inconsistent system**

A system of simultaneous linear equations is said to be inconsistent if it has no solution.

#### ❖ A pair of linear equations in two variables can be solved by

- 1) **Graphical method** or
- 2) **Algebraic method**

❖ **Nature of solution of simultaneous linear equations**

• **Based on graph:**

**Case (i):** The lines intersect at a point.

The point of intersection is the unique solution of the two equations.

In this case, the pair of equations is **consistent**.

**Case (ii):** The lines coincide

The pair of equations has infinitely many solutions – each point on the line is a solution. In this case, the pair of equations is dependent (which is **consistent**).

**Case (iii):** The lines are parallel.

The pair of equations has no solution. In this case, the pair of equations is **inconsistent**.

• **Based on the coefficients:**

Let  $a_1x + b_1y + c_1 = 0$ ,  $a_2x + b_2y + c_2 = 0$  be a system of linear equations.

**Case (i)**  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

In this case, the given system is consistent.

This implies that the system has a unique solution.

**Case (ii)**  $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

In this case, the given system is inconsistent.

This implies that the system has no solution.

**Case (ii)**  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

In this case, the given system is dependent and consistent.

This implies that the system has infinitely many solutions.

❖ **Algebraic method for solving simultaneous linear equations**

Simultaneous linear equations can be solved algebraically by the following methods.

• **Substitution method**

Example: Solve the following system of equations by substitution method.

$$x - 4y + 7 = 0$$

$$3x + 2y = 0$$

**Solution:**

The given equations are

$$x - 4y + 7 = 0 \quad \dots (1)$$

$$3x + 2y = 0 \quad \dots (2)$$

From equation (2),  $3x = -2y$



$$\Rightarrow x = -\frac{2}{3}y$$

Put  $x = -\frac{2}{3}y$  in equation (1)

$$-\frac{2}{3}y - 4y + 7 = 0$$

$$\Rightarrow \frac{-2y - 12y}{3} = -7$$

$$\Rightarrow -14y = -21$$

$$\Rightarrow y = \frac{-21}{-14} = \frac{3}{2}$$

$$\therefore x = -\frac{2}{3}\left(\frac{3}{2}\right) = -1$$

Therefore, the required solution is  $\left(-1, \frac{3}{2}\right)$ .

- **Elimination method**

Solve the following pair of linear equations by elimination method.

$$7x - 2y = 10$$

$$5x + 3y = 6$$

**Solution:**

$$7x - 2y = 10 \quad \dots (1)$$

$$5x + 3y = 6 \quad \dots (2)$$

Multiplying equation (1) by 5 and equation (2) by 7, we get

$$35x - 10y = 50 \quad \dots (3)$$

$$35x + 21y = 42 \quad \dots (4)$$

Subtracting equation (4) from (3), we get

$$-31y = 8 \Rightarrow y = -\frac{8}{31}$$

Now, using equation (1):

$$7x = 10 + 2y$$

$$\Rightarrow x = \frac{1}{7} \left\{ 10 + 2 \times \frac{-8}{31} \right\} = \frac{42}{31}$$

Required solution is  $\left(\frac{42}{31}, -\frac{8}{31}\right)$

- **Cross-multiplication method**

The solution of the system of linear equations  $a_1x + b_1y + c_1 = 0$ ,  $a_2x + b_2y + c_2 = 0$  can be determined by the following diagram.



That is,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \quad (a_1b_2 - a_2b_1 \neq 0)$$

- ❖ Certain pairs of equations that are not linear can be reduced to linear form by suitable substitutions.

**Example:** Consider the following system of equations.

$$\frac{2}{x-2} - \frac{1}{y-1} = 1$$

$$\frac{5}{x-2} - \frac{6}{y-1} = 20$$

Let  $x - 2 = u$ ,  $y - 1 = v$ . Then, the given system of equations reduces to

$$2u - v = 1$$

$$5u - 6v = 20$$

## Chapter 6: Triangles

### ❖ Congruent and similar figures

- Two geometric figures having the same shape and size are said to be congruent figures.
- Two geometric figures having the same shape, but not necessarily the same size, are called similar figures.
- All congruent figures are similar. However, the converse is not true.
- Two polygons with the same number of sides are similar, if
  - a) their corresponding angles are equal
  - b) their corresponding sides are in the same ratio (or proportion)

### ❖ Similarity of triangles

Two triangles are similar, if

- their corresponding angles are equal
- their corresponding sides are in the same ratio (or proportion)

### ❖ Basic proportionality theorem

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

**Corollary:** If D and E are points on the sides, AB and AC, respectively of  $\triangle ABC$  such that  $DE \parallel BC$ , then

a. 
$$\frac{AB}{AD} = \frac{AC}{AE}$$

b. 
$$\frac{AB}{DB} = \frac{AC}{EC}$$

❖ **Converse of basic proportionality theorem**

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

❖ **Criteria for similarity of triangles**

• **AAA similarity criterion**

If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence, the two triangles are similar.

• **AA similarity criterion**

If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar.

• **SSS similarity criterion**

If in two triangles, sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence, the two triangles are similar.

• **SAS similarity criterion**

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

❖ **Areas of similar triangles**

• The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

• If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

• **Pythagoras theorem**

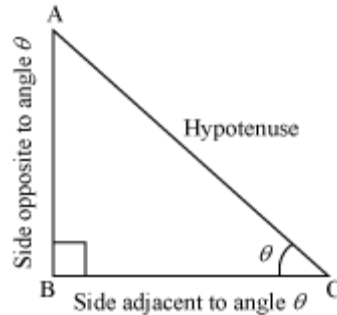
In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

• **Converse of Pythagoras theorem**

In a triangle, if the square of one side is equal to the sum of the squares of other two sides, then the angle opposite to the first side is a right angle.

## Chapter 8: Introduction to Trigonometry

### ❖ Trigonometric ratios



$$\sin \theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\cos \theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BC}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{AC}{AB}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{AC}{BC}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{BC}{AB}$$

$$\text{Also, } \tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta}$$

If one of the trigonometric ratios of an acute angle is known, then the remaining trigonometric ratios of the angle can be calculated.

### ❖ Trigonometric ratios of some specific angles

$\theta$	0	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined

$\operatorname{cosec} \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot \theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

❖ **Trigonometric ratios of complementary Angles**

$$\begin{aligned} \sin(90^\circ - \theta) &= \cos \theta & \cos(90^\circ - \theta) &= \sin \theta \\ \tan(90^\circ - \theta) &= \cot \theta & \cot(90^\circ - \theta) &= \tan \theta \\ \operatorname{cosec}(90^\circ - \theta) &= \sec \theta & \sec(90^\circ - \theta) &= \operatorname{cosec} \theta \end{aligned}$$

Where  $\theta$  is an acute angle.

❖ **Trigonometric identities**

- $\cos^2 A + \sin^2 A = 1$
- $1 + \tan^2 A = \sec^2 A$
- $1 + \cot^2 A = \operatorname{cosec}^2 A$

## Chapter 14: Statistics

❖ **Mean of ungrouped data**

If  $x_1, x_2, \dots, x_n$  are observations with respective frequencies  $f_1, f_2, \dots, f_n$  for a given data,

then the mean ( $\bar{x}$ ) of the data is given by  $\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \dots + f_n x_n}{f_1 + f_2 + \dots + f_n}$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}, \text{ where } i \text{ varies from } 1 \text{ to } n$$

❖ **Mean of grouped data**

- **Direct method**

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}, \text{ where } f_i \text{ is the frequency corresponding to the class mark } x_i.$$

- **Assumed-mean method**

$$\bar{x} = a + \bar{d} = a + \frac{\sum f_i d_i}{\sum f_i}, \text{ where 'a' is the assumed mean, } d_i = x_i - a, \text{ and } f_i \text{ is the}$$

frequency corresponding to the class mark  $x_i$

- **Step-deviation method**

$$\bar{x} = a + h\bar{u} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right), \text{ where } u_i = \frac{x_i - a}{h}, f_i \text{ is the frequency corresponding to}$$

the class mark  $x_i$ ,  $a$  is the assumed mean and  $h$  is the class size.

- The assumed-mean method and the step-deviation method are simplified forms of the direct method.
- The mean obtained by all the three methods is the same.
- Step-deviation method is convenient to apply if all  $d_i$ 's have a common factor.

#### ❖ Mode

- **Mode of ungrouped data**

The mode or modal value of a distribution is the observation for which the frequency is the maximum.

- **Mode of grouped data**

Mode of a grouped data is given by:

$$\text{Mode} = l \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

Where,  $l$  = Lower limit of the modal class

$h$  = Size of the class interval (assuming all class sizes to be equal)

$f_1$  = Frequency of the modal class

$f_0$  = Frequency of the class preceding the modal class

$f_2$  = Frequency of the class succeeding the modal class

#### ❖ Median

- **Median of ungrouped data**

If  $n$  (number of observations) is an odd number, then median = value of the  $\left(\frac{n+1}{2}\right)^{\text{th}}$  observation

If  $n$  (number of observations) is an even number, then median = mean of the values of

the  $\left(\frac{n}{2}\right)^{\text{th}}$  and  $\left(\frac{n}{2}+1\right)^{\text{th}}$  observations

- **Median of grouped data**

Median of a grouped data is given by:

$$\text{Median} = l + \left( \frac{\frac{n}{2} - cf}{f} \right) \times h$$

Where  $l$  = Lower limit of median class

$n$  = Number of observations

$cf$  = Cumulative frequency of the class preceding the median class

$f$  = Frequency of the median class

$h$  = Class size (assuming class size to be equal)

❖ **Empirical relationship between the three measures of central tendency**

$$3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$$

❖ **Graphical representation of cumulative frequency distribution**

- **Ogive (of the less- than type)**

Draw ogive of the less-than type for the given distribution.

Class interval	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	7	8	6	8	6	5

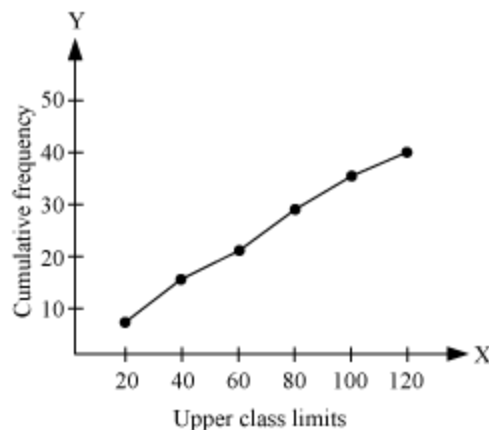
**Solution:**

The cumulative frequency distribution for the given data can be found as:

Class interval	Upper class limit	Frequency	Cumulative frequency
0 – 20	20	7	7
20 – 40	40	8	15
40 – 60	60	6	21
60 – 80	80	8	29
80 – 100	100	6	35
100 – 120	120	5	40

By taking the horizontal axis as the upper class limit and the vertical axis as the corresponding cumulative frequency, we can plot the cumulative frequency for each upper class limit.

Then, the required ogive (of the less-than type) is obtained as:



- **Ogive (of the more-than type)**

Example: Draw ogive of the more-than type for the following distribution.

Class interval	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	7	8	6	8	6	5

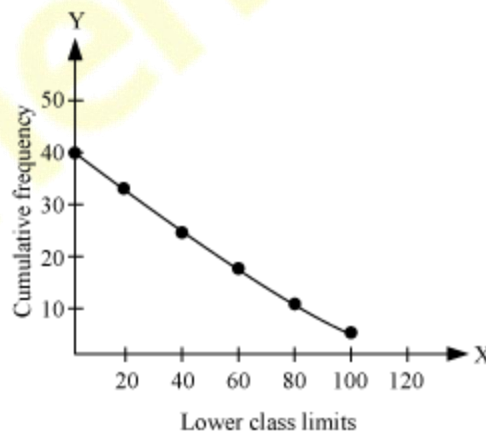
**Solution:**

The cumulative frequency for the given data can be found as:

Class interval	Lower class limit	Frequency	Cumulative frequency
0 – 20	0	7	40
20 – 40	20	8	33
40 – 60	40	6	25
60 – 80	60	8	19
80 – 100	80	6	11
100 – 120	100	5	5

By taking the horizontal axis as the lower class limit and the vertical axis as the corresponding cumulative frequency, we can plot the cumulative frequency for each lower class limit.

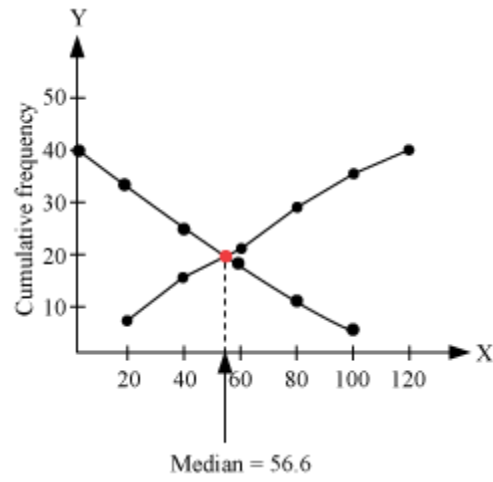
Then, the required ogive (of the more-than type) is obtained as:



**Note:**

The x-coordinate of the point of intersection of the “more-than ogive” and “less-than ogive” of a given grouped data gives its median.





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