Synopsis – Grade 10 Math Term II

Chapter 4: Quadratic Equations

- **General form of quadratic equations**

  The general form of quadratic equation in the variable 'x' is $ax^2+bx+c=0$, where a, b, c are real numbers and $a \neq 0$.

  For example, $3x^2+6x+2=0$, $x^2-2=0$

- **Roots of quadratic equations**

  A real number 'k' is said to be the root of the quadratic equation, $ax^2+bx+c=0$, if $ak^2+bk+c=0$

  For example, 3 and –10 are the roots of the quadratic equation, $x^2+7x-30=0$, because

  $3^2+7\times3-30=9+21-30=30-30=0=$ R.H.S.

  $(-10)^2+7\times(-10)-30=100-70-30=0=$ R.H.S.

  Note: $x = \alpha$ (α may or may not be real) is a solution of the quadratic equation, $ax^2+bx+c=0$, if it satisfies the quadratic equation.

- **Solution of quadratic equation by factorization method**

  If we can factorise $ax^2+bx+c=0$, where $a \neq 0$, into a product of two linear factors, then the roots of this quadratic equation can be calculated by equating each factor to zero.

  Example: Find the roots of the equation, $2x^2-7\sqrt{3}x+15=0$, by factorisation.

  Solution:

  $2x^2-7\sqrt{3}x+15=0$

  $\Rightarrow 2x^2-2\sqrt{3}x-5\sqrt{3}x+15=0$

  $\Rightarrow 2x\left(x-\sqrt{3}\right)-5\sqrt{3}\left(x-\sqrt{3}\right)=0$

  $\Rightarrow \left(x-\sqrt{3}\right)\left(2x-5\sqrt{3}\right)=0$

  $\left(x-\sqrt{3}\right)=0$ or $\left(2x-5\sqrt{3}\right)=0$

  $\Rightarrow x=\sqrt{3}$ or $x=\frac{5\sqrt{3}}{2}$

  Therefore, $\sqrt{3}$ and $\frac{5\sqrt{3}}{2}$ are the roots of the given quadratic equation.

- **Solution of quadratic equation by completing the square**

  A quadratic equation can also be solved by the method of completing the square.

Example: Find the roots of the quadratic equation, $5x^2+7x-6=0$, by the method of completing the square.

Solution:

$$5x^2+7x-6=0$$

$$\Rightarrow 5\left[x^2+\frac{7}{5}x-\frac{6}{5}\right]=0$$

$$\Rightarrow x^2+2\cdot x\cdot\frac{7}{10}+\left(\frac{7}{10}\right)^2-\left(\frac{7}{10}\right)^2-\frac{6}{5}=0$$

$$\Rightarrow \left(x+\frac{7}{10}\right)^2-\frac{49}{100}-\frac{6}{5}=0$$

$$\Rightarrow \left(x+\frac{7}{10}\right)^2=\frac{169}{100}$$

$$\Rightarrow \left(x+\frac{7}{10}\right)=\pm\sqrt{\frac{169}{100}}=\pm\frac{13}{10}$$

$$x+\frac{7}{10}=\frac{13}{10} \text{ or } x+\frac{7}{10}=\frac{-13}{10}$$

$$\Rightarrow x=\frac{13}{10}-\frac{7}{10}=\frac{3}{5} \text{ or } x=\frac{-13}{10}-\frac{7}{10}=-2$$

Therefore, –2 and $\frac{3}{5}$ are the roots of the given quadratic equation.

❖ **Quadratic formula**

The roots of the quadratic equation, $ax^2+bx+c=0$, are given by,

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, where $b^2-4ac\geq 0$

❖ **Nature of roots of quadratic equations**

For the quadratic equation, $ax^2+bx+c=0$, where $a\neq 0$, the discriminant 'D' is defined as

$$\mathrm{D}=b^2-4ac$$

The quadratic equation, $ax^2+bx+c=0$, where $a\neq 0$, has

(i) two distinct real roots, if $\mathrm{D}=b^2-4ac>0$

(ii) two equal real roots, if $\mathrm{D}=b^2-4ac=0$

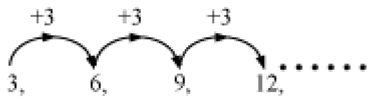
(iii) no real roots, if $\mathrm{D}=b^2-4ac<0$

Chapter 5: Arithmetic Progressions

❖ **Arithmetic progression (AP)**

- An arithmetic progression is a list of numbers in which the difference between any two consecutive terms is equal.
- In an AP, each term, except the first term, is obtained by adding a fixed number called common difference to the preceding term.
- The common difference of an AP can be positive, negative or zero.

  Example:

  is an AP whose first term and common difference are 3 and 3 respectively.

❖ **General form of AP**

- The general form of an AP can be written as $a, a + d, a + 2d, a + 3d$ …, where a is the first term and d is the common difference.
- A given list of numbers i.e., a_1, a_2, a_3 … forms an AP if $a_{k+1} - a_k$ is the same for all values of k.

❖ **n^{th} term of an AP**

The n^{th} term (a_n) of an AP with first term a and common difference d is given by

$a_n = a + (n - 1) d$

Here, a_n is called the general term of the AP.

❖ **Sum of first n terms of an AP**

The sum of the first n terms of an AP is given by

$S = \frac{n}{2}\left[2a + (n-1)d\right]$, where a is the first term and d is the common difference.

If there are only n terms in an AP, then $S = \frac{n}{2}\left[a + d\right]$, where $d = a_n$ is the last term.

Chapter 7: Coordinate Geometry

❖ **Axes and coordinates**

- The distance of a point from the y-axis is called its x-coordinate or abscissa, and the distance of the point from the x-axis is called its y-coordinate or ordinate.
- If the abscissa of a point is x and the ordinate is y, then (x, y) are called the coordinates of the point.
- The coordinates of a point on the x-axis are of the form $(x, 0)$ and the coordinates of the point on the y-axis are of the form $(0, y)$.
- The coordinates of the origin are (0, 0).
- The coordinates of a point are of the form (+, +) in the first quadrant, (–, +) in the second quadrant, (–, –) in the third quadrant and (+, –) in the fourth quadrant, where + denotes a positive real number and – denotes a negative real number.

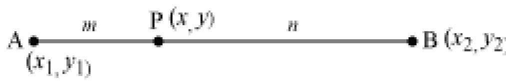
❖ **Distance formula**

The distance between the points P(x_1, y_1) and Q (x_2, y_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Cor: The distance of a point (x, y) from the origin O (0, 0) is given by $OP = \sqrt{x^2 + y^2}$.

❖ **Section formula**



The co-ordinates of the point P (x, y), which divides the line segment joining the points A (x_1, y_1) and B (x_2, y_2) internally in the ratio m:n, are given by:

$$P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Cor: The mid-point of the line segment joining the points A (x_1, y_1) and B (x_2, y_2) is

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ [Note: Here, $m = n = 1$]

❖ **Area of a triangle**

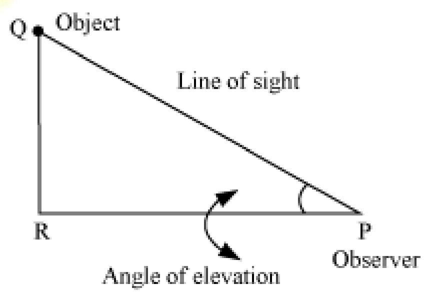
The area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by the numerical value of the expression $\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

Chapter 9: Some Applications of Trigonometry

❖ **Line of sight**

It is the line drawn from the eye of an observer to a point on the object viewed by the observer.

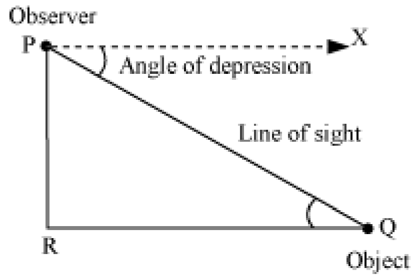
❖ **Angle of elevation**



Let P be the position of the eye of the observer. Let Q be the object above the horizontal line PR.

Angle of elevation of the object Q with respect to the observer P is the angle made by the line of sight PQ with the horizontal line PR. That is, ∠QPR is the angle of elevation.

❖ **Angle of depression**



Let P be the position of the eye of the observer. Let Q be the object below the horizontal line PX.

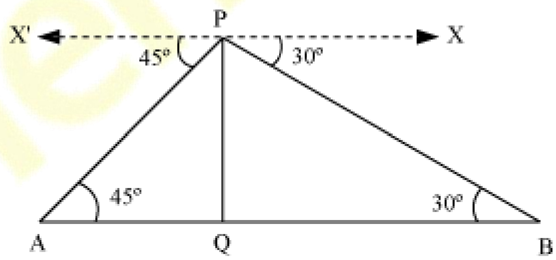
Angle of depression of the object Q with respect to the observer P is the angle made by the line of sight PQ with the horizontal line PX. That is, ∠XPQ is the angle of depression. It can be seen that

∠PQR = ∠XPQ [Alternate interior angles]

❖ The height or length of an object or the distance between two distant objects can be calculated by using trigonometric ratios.

Example: Two wells are located on the opposite sides of a 18 m tall building. As observed from the top of the building, the angles of depression of the two wells are 30° and 45°. Find the distance between the wells. [Use $\sqrt{3} = 1.732$]

Solution: The given situation can be represented as



Here, PQ is the building. A and B are the positions of the two wells such that:

∠XPB = 30°, ∠X′PA =45°

Now, ∠PAQ = ∠X′PA = 45°

∠PBQ = ∠XPB = 30°

In ΔPAQ, we have

$$\frac{PQ}{AQ} = \tan 45°$$

$$\Rightarrow \frac{18}{AQ} = 1$$

$$\Rightarrow AQ = 18\,m$$

In ΔPBQ, we have

$$\frac{PQ}{QB} = \tan 30°$$

$$\Rightarrow \frac{18}{QB} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow QB = 18\sqrt{3}$$

$$\therefore AB = AQ + QB = \left(18 + 18\sqrt{3}\right) m$$

$$= 18\left(1 + \sqrt{3}\right) m$$

$$= 18\left(1 + 1.732\right) m$$

$$= 18 \times 2.732 \text{ m}$$

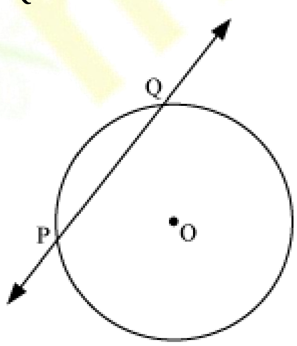
$$= 49.176 \text{ m}$$

Chapter 10: Circles

- **Secant**

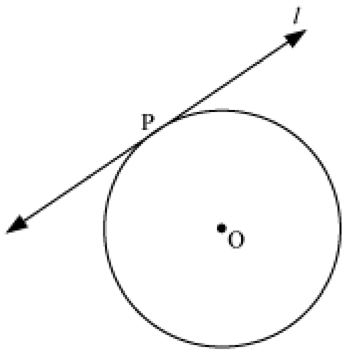
  A line that intersects a circle in two points is called a secant of the circle.

  Here, $\overleftrightarrow{PQ}$ is a secant of the circle with centre O.



- **Tangent**

  A tangent to a circle is a line that intersects the circle at exactly one point. The point is called the point of contact of the tangent.



Here, *l* is tangent to the circle with centre O and point P is the point of contact of the tangent *l*.

- Only one tangent can be drawn at a point on the circle.
- The tangent to a circle is a particular case of the secant, when the two end points of its corresponding chord coincide.

❖ The tangent at any point on a circle is perpendicular to the radius through the point of contact.

- No tangent can be drawn to a circle passing through a point lying inside the circle.
- One and only one tangent can be drawn to a circle passing through a point lying on the circle.
- Exactly two tangents can be drawn to a circle through a point lying outside the circle.

❖ **Length of the tangent**

The length of the segment of the tangent from an external point P to the point of contact with the circle is called the length of the tangent from the point P to the circle.

❖ The lengths of tangents drawn from an external point to a circle are equal.

Chapter 11: Constructions

❖ **Division of a line segment in a given ratio**

Example: Draw $\overline{PQ} = 9$ cm and divide it in the ratio 2:5.

Steps of construction:

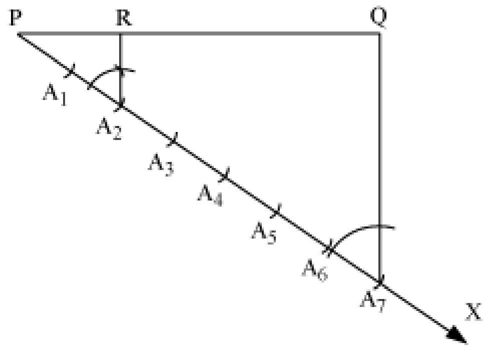
(1) Draw $\overline{PQ} = 9$ cm

(2) Draw a ray $\overrightarrow{PX}$, making an acute angle with PQ.

(3) Mark 7 (= 2 + 5) points A_1, A_2, A_3 … A_7 along PX such that
$PA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$

(4) Join QA_7

(5) Through the point A_2, draw a line parallel to QA_7 by making an angle equal to $\angle PQA_7$ at A_2, intersecting PQ at point R.
$\therefore PR:RQ = 2:5$



❖ Construction of a triangle similar to a given triangle as per the given scale factor

- **Case I: Scale factor less than 1**

  Example: Draw a ΔABC with sides BC = 8 cm, AC = 7 cm, and ∠B = 70°. Then, construct a similar triangle whose sides are $\left(\frac{3}{5}\right)^{th}$ of the corresponding sides of ΔABC.

  Steps of construction:

  (1) Draw BC = 8 cm

  (2) At B, draw ∠XBC = 70°

  (3) With C as centre and radius 7 cm, draw an arc intersecting BX at A.

  (4) Join AB, and ΔABC is thus obtained.

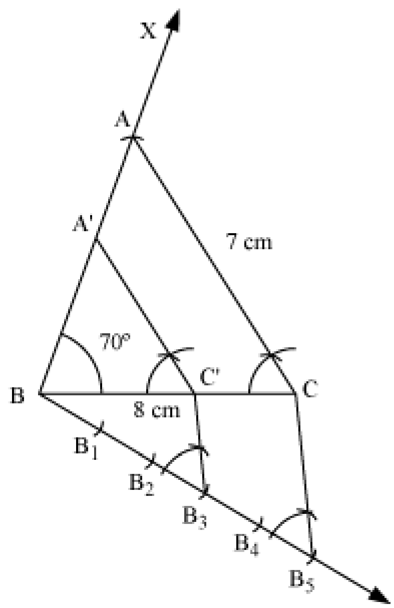
  (5) Draw a ray $\overrightarrow{BY}$, making an acute angle with BC.

  (6) Mark 5 points, B_1, B_2, B_3, B_4, B_5, along BY such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$

  (7) Join CB_5

  (8) Through the point B_3, draw a line parallel to B_5C by making an angle equal to $\angle BB_5C$, intersecting BC at C′.

  (9) Through the point C′, draw a line parallel to AC, intersecting BA at A′. Thus,



$\Delta A'BC'$ is the required triangle.

- **Case II: Scale factor greater than 1**

Example: Construct an isosceles triangle with base 5 cm and equal sides of 6 cm. Then, construct another triangle whose sides are $\left(\frac{4}{3}\right)^{rd}$ of the corresponding sides of the first triangle.

Steps of construction:

(1) Draw BC = 5 cm

(2) With B as the centre and C as the radius 6 cm, draw arcs on the same side of BC, intersecting at A.

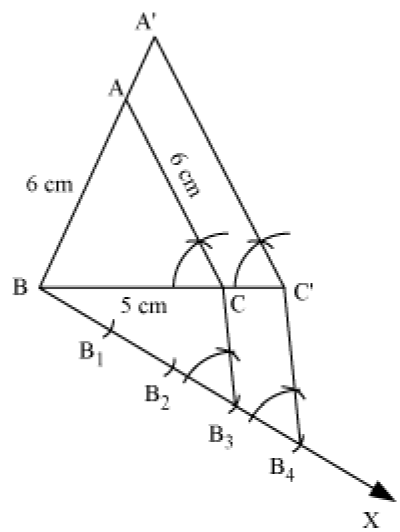
(3) Join AB and AC to get the required ΔABC.

(4) Draw a ray $\overrightarrow{BX}$, making an acute angle with BC on the side opposite to the vertex A.

(5) Mark 4 points B_1, B_2, B_3, B_4, along BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

(6) Join B_3C. Draw a line through B_4 parallel to B_3C, making an angle equal to $\angle BB_3C$, intersecting the extended line segment BC at C'.

(7) Through point C', draw a line parallel to CA, intersecting extended BA at A'.



The resulting $\Delta A'BC'$ is the required triangle.

- **Construction of tangents to a circle**

  Example: Draw a circle of radius 3 cm. From a point 5 cm away from its centre, construct a pair of tangents to the circle and measure their lengths.

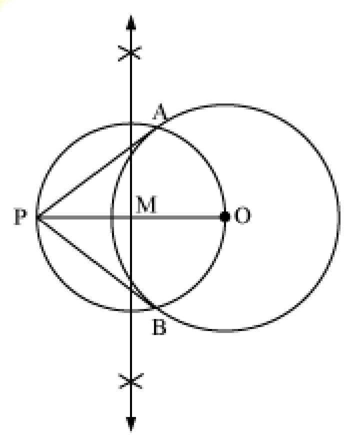
  Steps of construction:

  (1) First draw a circle with centre O and radius 3 cm. Take a point P such that OP = 5 cm, and then join OP.

  (2) Draw a perpendicular bisector of OP. Let M be the mid point of OP.

  (3) With M as the centre and OM as the radius, draw a circle. Let it intersect the previously drawn circle at A and B.

  (4) Joint PA and PB. Therefore, PA and PB are the required tangents. It can be observed that PA = PB = 4 cm.



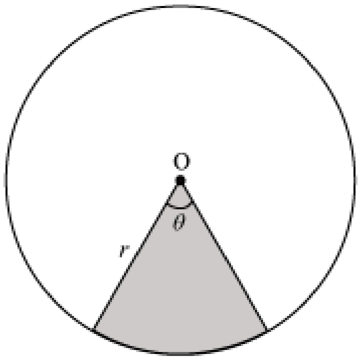
Chapter 12: Areas Related to Circles

- Area of a circle = πr^2

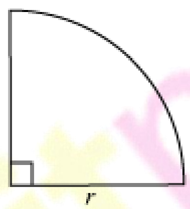
Circumference of a circle = $2\pi r$; where r is the radius of a circle.

- **Area of sector of a circle**

  - Area of the sector of angle $\theta = \frac{\theta}{360°} \times \pi r^2$, where r is the radius of the circle

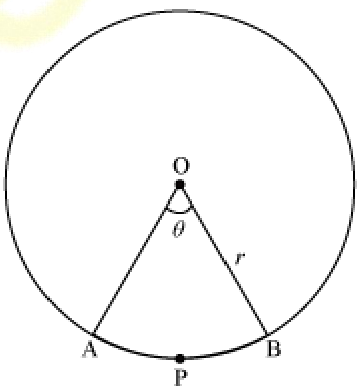


  - Area of a quadrant of a circle with radius $r = \frac{\pi r^2}{4}$ $[\because$ For quadrant $\theta = 90°]$



- **Length of arc of a circle**

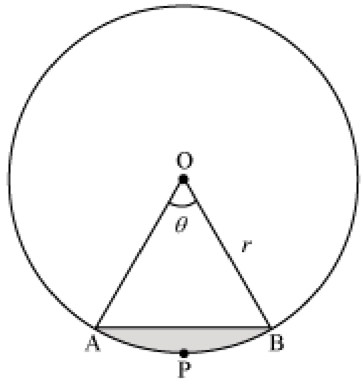
Length of the arc of a sector of angle $\theta = \frac{\theta}{360°} \times 2\pi r$, where r is the radius of the circle



- **Area of segment of a circle**

Area of segment APB = Area of sector OAPB − Area of ΔOAB

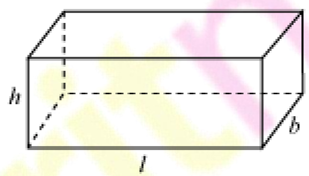
$$= \frac{\theta}{360} \times \pi r^2 - \text{area of } \Delta\text{OAB}$$



Chapter 13: Surface Areas and Volumes

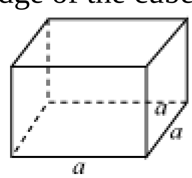
❖ **Cuboid**

- Surface area = 2 (lb + bh + hl)
- Volume = $l \times b \times h$, where l, b, h are respectively length, breadth and height of the cuboid



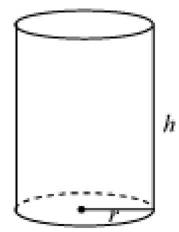
❖ **Cube**

- Surface area = $6a^2$
- Volume = a^3, where a is the edge of the cube



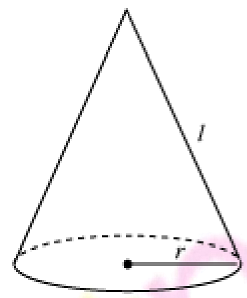
❖ **Cylinder**

- Curved surface area (CSA) = $2\pi rh$
- Total surface area (TSA) $= 2\pi r^2 + 2\pi rh = 2\pi r\left(r + h\right)$
- Volume = $\pi r^2 h$, where r is the radius and h is the height of the cylinder



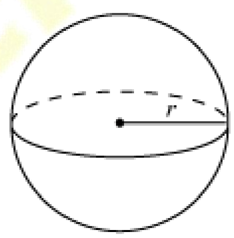
❖ Cone

- Curved surface area (CSA) = πrl
- Total surface area (TSA) $= \pi r^2 + \pi rl = \pi r\left(r + l\right)$
- Volume $= \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height of the cone



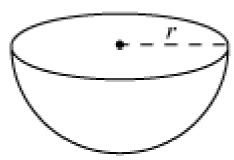
❖ Sphere

- Surface area = $4\pi r^2$
- Volume = $\frac{4}{3}\pi r^3$, where r is the radius of the sphere



❖ Hemisphere

- Curved surface area (CSA) = $2\pi r^2$
- Total surface area (TSA) = $3\pi r^2$
- Volume = $\frac{2}{3}\pi r^3$, where r is the radius of the hemisphere



Note: Volume of the combination of solids is the sum of the volumes of the individual solids

❖ **Conversion of a solid from one shape into another**

When a solid is converted into another solid of a different shape, the volume of the solid does not change.

❖ **Frustum of a cone**

- Volume of the frustum of a cone $= \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1\, r_2\right)h$
- CSA of the frustum of a cone $= \pi\left(r_1\ r_2\right)l$, where $l = \sqrt{h^2 + \left(r_1 - r_2\right)^2}$
- TSA of the frustum of a cone $= \pi\left(r_1 + r_2\right)l + \pi r_1^2 + \pi r_2^2$, where $l = \sqrt{h^2 + \left(r_1 - r_2\right)^2}$

Chapter 15: Probability

❖ The theoretical probability (also called classical probability) of an event E, denoted as $P(E)$ is given by

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Number of all possible outcomes of the experiment}}$$

❖ **Elementary events:**

- An event having only one outcome of the experiment is known as elementary event.
- The sum of the probabilities of all the elementary events of an experiment is 1.

Example: A dice is thrown once. What is the probability of getting 1 on the dice?

Solution:

When a dice is thrown once, the possible outcomes are 1, 2, 3, 4, 5, 6.

Let A be the event of getting 1 on the dice.

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to } A}{\text{Number of all possible outcomes}} = \frac{1}{6}$$

❖ **Complementary events**

For an event E of an experiment, the event $\overline{E}$ represents 'not E', which is called the complement of the event E. We say, E and $\overline{E}$ are **complementary** events.

$$P(E) + P(\overline{E}) = 1$$

$$\Rightarrow \mathrm{P}(\overline{E}) = 1 - P(E)$$

- The probability of an impossible event of an experiment is 0.
- The probability of a sure (or certain) event of an experiment is 1.

  $\therefore 0 \le P(E) \le 1$